Market Capitalization and Equity Tail Risk: Empirical Calibration of Stress-Test Buckets for Portfolio Margin

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Abstract

Portfolio margin stress testing in the United States applies uniform shock magnitudes regardless of the underlying equity's market capitalization, despite decades of evidence that small-capitalization stocks exhibit greater downside risk. We construct the first empirical calibration of market-cap-differentiated stress-test buckets using 5.37 million return observations across 2,509 validated US equities over a 21-year period (2005--2026). Using properly computed historical market capitalizations derived from SEC-filed shares outstanding, and applying a cross-source validation filter against SEC EDGAR XBRL filings, we partition the cross-section into eight equal-observation quantile buckets and measure 3-day Value-at-Risk (99th percentile) and Conditional Value-at-Risk. We find a strictly monotonic relationship between market capitalization and tail risk: 3-day cVaR at the 99.9th percentile ranges from -50% for micro-caps (below $280 million) to -27% for large-caps (above $23 billion). The CVaR-to-VaR ratio is remarkably stable at 1.41--1.45 across all buckets, indicating uniformly fat-tailed distributions relative to the Gaussian benchmark of 1.33. The monotonic relationship is robust to alternative bucket counts, sub-period analysis, and filter specifications. We propose an eight-tier stress grid that substantially tightens requirements for mid- and large-cap equities relative to current practice, while modestly increasing micro-cap stress levels. We also document the iterative human-AI research methodology that produced these results, offering a case study in collaborative empirical finance.

Keywords: portfolio margin, stress testing, market capitalization, tail risk, Value-at-Risk, Conditional Value-at-Risk, size effect, AI collaboration

JEL Classification: G11, G12, G17, G28

1. Introduction

The Options Clearing Corporation's (OCC) System for Theoretical Analysis and Numerical Simulations (STANS) applies a uniform scan range of +/-15% to all US equity positions when computing theoretical portfolio margin requirements. The Financial Industry Regulatory Authority's Rule 4210 sets a 15% minimum stress level

without mandating differentiation by issuer characteristics. In practice, prime brokers and clearing firms layer proprietary adjustments atop the regulatory baseline, but no published empirical framework maps equity market capitalization to calibrated stress-test magnitudes.

This gap is surprising given the depth of the academic literature on the size effect. Since Banz (1981) documented that small-capitalization firms earn risk-adjusted excess returns, and Fama and French (1993) formalized the size factor (SMB) in their three-factor model, the relationship between firm size and equity volatility has been well established. Ang, Chen, and Xing (2006) extended this insight to downside risk, demonstrating that downside beta---the sensitivity of returns to market declines---is asymmetrically larger for small-cap stocks. Horta, Mendes, and Vieira (2010) confirmed that small-cap exchange-traded funds experience disproportionately larger losses during extreme market downturns than the gains they capture during upswings.

Despite this body of evidence, the translation from "small stocks are riskier" to "how much riskier, precisely, for margin stress-testing purposes" has not been made in the literature. Clearing houses and prime brokers rely on proprietary models or judgment-based grids that are not publicly calibrated. The contribution of this paper is threefold.

First, we construct, to our knowledge, the first empirical mapping from market capitalization to discrete stress-test buckets calibrated at the 99th and 99.9th percentiles of the 3-day return distribution. Our recommended grid is monotonic, robust, and directly implementable in portfolio margin systems.

Second, we address a methodological challenge that has likely deterred prior work: the correct computation of historical market capitalization. Naive approaches using adjusted stock prices and current shares outstanding produce severely distorted historical market caps due to the compounding of split and dividend adjustments over long horizons. We document this failure mode and present a solution using actual quarterly shares outstanding from SEC filings, available through the yfinance `get_shares_full()` function.

Third, we document the iterative research process through which these results were produced, involving close collaboration between a human researcher and an AI system (Claude, developed by Anthropic). The AI served not merely as a computational tool but as a diagnostic partner, identifying data quality problems, proposing alternative methodologies, and iterating toward robust results. We present this process in Appendix A as a case study in human-AI research collaboration.

The remainder of the paper proceeds as follows. Section 2 reviews the relevant literature on the size effect, downside risk, and clearing house stress-testing practices. Section 3 describes our data, the historical market capitalization methodology, and our

approach to bucket construction and tail risk measurement. Section 4 presents the main results, including the eight-bucket stress grid and quantile regression analysis. Section 5 reports robustness tests across filter specifications, bucket counts, sub-periods, and alternative lookback windows. Section 6 discusses practical implications, limitations, and the AI-guided methodology. Section 7 concludes.

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2. Literature Review

2.1 The Size Effect and Equity Volatility

The size effect---the empirical regularity that smaller firms earn higher average returns and exhibit higher volatility---is one of the most extensively documented phenomena in asset pricing. Banz (1981) first identified the negative cross-sectional relationship between market capitalization and risk-adjusted returns on the New York Stock Exchange. Fama and French (1993) formalized this relationship through the SMB (Small Minus Big) factor in their three-factor model, establishing that a portfolio long small-cap and short large-cap stocks earns a positive risk premium. Subsequent work by Fama and French (2015) retained the size factor in their five-factor specification, underscoring its persistence.

The risk-based explanation for the size premium centers on the higher systematic and idiosyncratic volatility of small-cap equities. Smaller firms tend to have less diversified revenue streams, higher leverage ratios, thinner analyst coverage, and lower liquidity---all of which amplify return dispersion (Amihud, 2002; Pastor and Stambaugh, 2003).

2.2 Downside Risk and Asymmetry

The relevance of the size effect to stress testing is sharpened by evidence on asymmetric downside risk. Ang, Chen, and Xing (2006) demonstrated that the cross-section of expected returns is better explained by downside beta---the covariance of individual returns with market returns conditional on the market declining---than by unconditional beta. They found that stocks with high downside beta earn a significant premium, and that this premium is not explained by standard risk factors including size.

Horta, Mendes, and Vieira (2010) examined the tail dependence structure of small- and large-cap ETFs using copula methods, finding that small-cap portfolios exhibit greater lower-tail dependence with the market. In practical terms, small-cap stocks fall more than large-caps in extreme downturns, and this asymmetry is not fully captured by symmetric risk measures such as standard deviation.

Bali and Cakici (2004) studied the relationship between firm size and VaR, documenting that VaR sorted by market capitalization reveals a monotonic gradient consistent with the size premium being compensation for tail risk rather than mere volatility.

2.3 Clearing House and Regulatory Approaches

The OCC's STANS methodology, the regulatory baseline for US options clearing, employs a full-revaluation Monte Carlo framework that draws correlated asset returns from an estimated multivariate distribution (OCC, 2020). While STANS incorporates volatility clustering and fat tails, its published scan ranges for the simpler TIMS (Theoretical Intermarket Margining System) framework apply a uniform +/-15% for equities and asymmetric ranges (-8%/+6%) for broad-based indices. No published documentation describes market-cap-differentiated scan ranges within STANS.

FINRA Rule 4210 requires a minimum 15% portfolio margin stress test across at least ten equidistant scan points, without prescribing differentiation by issuer size. Individual broker-dealers may impose additional requirements under their risk management frameworks, but these proprietary overlays are not published.

In Europe, the European Securities and Markets Authority's (ESMA) guidelines on central counterparty margin models require "adequate coverage of exposures with a high degree of confidence" but similarly do not mandate size-based calibration for equity portfolios (ESMA, 2018).

2.4 The Gap

To our knowledge, no published study has (a) empirically measured the 3-day tail risk of US equities conditional on market capitalization at percentiles relevant to margin stress testing (99th and beyond), (b) proposed a calibrated set of discrete stress-test buckets mapping from cap ranges to shock magnitudes, or (c) tested the robustness of such a mapping across sub-periods, bucket specifications, and filter choices. This paper addresses all three.

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3. Data and Methodology

3.1 Universe and Sample

Our equity universe targets the broad US market by combining the constituents of three Russell/S&P indices: the S&P 500, the S&P MidCap 400, and the Russell 2000. After removing duplicates, this yields approximately 2,900 unique tickers representing the full market-capitalization spectrum from micro-cap to mega-cap.

We retrieve daily price data from Yahoo Finance via the yfinance Python library for the period May 2005 through March 2026, a span of approximately 21 years covering multiple complete market cycles including the Global Financial Crisis (2007--2009), the European sovereign debt crisis (2011--2012), the COVID-19 crash (2020), and the 2022 monetary tightening episode.

After applying data quality filters (described in Section 3.3) and cross-source share count validation against SEC EDGAR (described in Section 3.2), our final sample comprises 5,371,525 daily return observations across 2,509 validated equities.

3.2 Historical Market Capitalization

The correct computation of historical market capitalization is central to this study and presents a methodological challenge that we believe has contributed to the absence of prior work on this topic.

The naive approach and its failure. The most intuitive method for computing historical market capitalization is to multiply the current number of shares outstanding by the historical stock price. When using financial data providers such as Yahoo Finance, the default price series is the "adjusted close," which retroactively modifies historical prices to account for stock splits, reverse splits, and dividend distributions. This adjustment is designed to produce a continuous return series for performance measurement---not for reconstructing historical market values.

The problem is severe. Consider a company that has undergone multiple forward splits over 20 years (e.g., 2:1 splits in 2007, 2014, and 2020, yielding a cumulative 8:1 adjustment). Its adjusted price in 2005 will be one-eighth of the actual trading price, while the current shares outstanding reflect all post-split expansion. Multiplying these two numbers produces a historical market cap that is correct in neither direction: it understates the price by 8x while using the fully expanded share count.

The distortions we observed using this naive approach were dramatic. Apple Inc. appeared to have a market capitalization of approximately $133 million in 2000 (actual: roughly $5 billion). Citigroup, due to its 1:10 reverse split in 2011, appeared at approximately $5 trillion in the pre-crisis period (actual: roughly $250 billion). Across the full sample, 96.5% of tickers appeared in multiple market-cap buckets over time---a plausible finding in principle, but at magnitudes that reflected data artifacts rather than genuine migration. Most critically, the resulting tail-risk analysis produced an inverted relationship at the top of the capitalization spectrum: the >= $1 trillion bucket showed worse tail risk than micro-caps, driven entirely by the distorted inclusion of mid-cap companies misclassified as mega-caps.

The solution: SEC-filed shares outstanding. Yahoo Finance's `get_shares_full()` function provides a time series of actual quarterly shares outstanding as reported in SEC filings (10-K and 10-Q). This series is available for most US-listed equities going back approximately 10 years (to around 2015 for many companies, and as far back as 2010 for others). By multiplying this actual share count by the unadjusted (raw) daily closing price, we obtain a historical market capitalization that correctly reflects the company's size as it was known to investors at the time.

The tradeoff is a shorter effective lookback for market-cap assignment: while we have daily returns going back to 2005, we can only assign market capitalizations for the period covered by `get_shares_full()` data. Observations without a valid historical market cap are excluded from the bucket analysis. This reduces our usable sample but eliminates the systematic distortions that would otherwise invalidate the results.

Cross-source validation. To verify that Yahoo Finance's `get_shares_full()` produces reliable historical market capitalizations, we conducted an independent validation against two external sources: daily closing prices from Financial Modeling Prep (FMP) and quarterly shares outstanding from SEC EDGAR's XBRL company facts API. For each validation observation, we computed an independent market capitalization as the product of the FMP closing price and the EDGAR-reported shares outstanding, then compared this to the Yahoo-derived market cap (unadjusted close × `get_shares_full()` shares).

The validation covers 50 randomly selected tickers from the study universe, yielding 1,536 quarterly observations spanning 2016--2026. Table 2a summarizes the results.

Table 2a: Cross-Source Market Capitalization Validation (Yahoo Finance vs. FMP + SEC EDGAR)

Statistic	Value
Tickers validated	50
Quarterly observations	1,536
Mean ratio (Yahoo / Independent)	1.0013
Median ratio	1.0006
5th--95th percentile range	0.9435 -- 1.0744
Mean absolute deviation	3.22%
Median absolute deviation	1.60%
Within ±2%	58.1%
Within ±5%	85.2%
Within ±10%	94.3%

The median ratio of 1.0006 indicates near-zero systematic bias between the two approaches. The residual dispersion (median absolute deviation of 1.60%) is attributable to minor differences in the exact reporting date of shares outstanding between Yahoo's `get_shares_full()` and EDGAR filings, as well as occasional intraday price differences between data providers. Critically, 85% of observations agree within 5%, and 94% within 10%---well within the tolerance required for bucket assignment, where the narrowest bucket spans a 2--3x range in market capitalization. Deviations at this scale cannot cause a misclassification that would affect the tail-risk estimates.

3.3 Return Computation and Filters

We compute 1-day simple returns as:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where P_t is the unadjusted closing price. For 3-day overlapping returns, we compute:

$$r_{t,t+3} = \frac{P_{t+3} - P_t}{P_t}$$

Overlapping returns are used to increase statistical power for the 3-day horizon, which is the relevant holding period for portfolio margin stress testing (representing a plausible forced-liquidation window for concentrated equity positions).

We apply three filters to the raw return data:

1. **Extreme return filter:** Observations with |1-day return| > 80% are excluded. This removes 514 observations (0.009% of the sample), which represent data errors, corporate actions not properly reflected in the price series, or moves so extreme as to be unrepeatable (e.g., post-restructuring re-listings). We test sensitivity to this filter in Section 5.1.

2. **Penny stock filter:** Observations where the closing price is below $1.00 are excluded. Penny stocks are typically not marginable and exhibit return dynamics (bid-ask bounce, manipulation) that are not representative of the marginable equity universe.

3. **Zero-volume filter:** Days with zero reported trading volume are excluded, as the closing price on such days does not reflect a market-clearing transaction.

3.4 Bucket Construction

The equal-observation quantile approach. We partition our market-capitalization-tagged observations into buckets using quantiles of the market-cap distribution, such

that each bucket contains approximately the same number of observations. This approach has three advantages over fixed, arbitrary breakpoints:

First, it ensures adequate statistical power in every bucket. Fixed breakpoints (e.g., $200 billion to $1 trillion) can produce buckets with very few observations, particularly at the extremes of the distribution, leading to noisy tail-risk estimates dominated by a handful of volatile names.

Second, it lets the data determine the natural breakpoints. Our top-octile boundary of $23 billion, for example, is not an ex ante judgment but the 87.5th percentile of the market-cap distribution of observation-days. This boundary separates a well-populated large-cap group from the rest of the distribution.

Third, it facilitates comparisons across different bucket counts (4, 5, 6, 8, 10, 12), as each specification is simply a different quantile partition of the same underlying distribution.

For our primary specification, we use eight buckets (octiles), yielding approximately 671,000 observations per bucket. The resulting boundaries and observation counts are reported in Table 1.

3.5 Tail Risk Measures

For each bucket, we compute three measures of left-tail risk on the distribution of 3-day returns:

1. **Value-at-Risk (VaR) at the 99th percentile:** The 1st percentile of the empirical return distribution. This is the loss level exceeded by 1% of observations.

2. **Conditional Value-at-Risk (CVaR) at the 99th percentile:** The mean of all returns below the 99th-percentile VaR. Also known as Expected Shortfall (ES), CVaR captures the average severity of losses in the worst 1% of outcomes.

3. **CVaR at the 99.9th percentile:** The mean of returns below the 0.1st percentile, capturing the most extreme tail events.

We also compute the CVaR/VaR ratio for each bucket, which provides a diagnostic for tail shape. Under a Gaussian distribution, the 99th-percentile CVaR/VaR ratio is approximately 1.33. Ratios substantially above this indicate fat tails; ratios that vary across buckets would indicate differential tail shape by market capitalization.

3.6 Quantile Regression

To complement the bucket analysis with a continuous specification, we estimate quantile regressions of the form:

$$Q_\tau(r_i) = \alpha_\tau + \beta_\tau \cdot \ln(\text{MarketCap}_i)$$

at $\tau \in {0.01, 0.023, 0.05}$, corresponding to the 1st, 2.3rd, and 5th percentiles of the 1-day return distribution. The 2.3rd percentile corresponds to approximately -2 standard deviations under normality. A positive β_τ indicates that larger market capitalization is associated with a less negative (less severe) conditional quantile---that is, smaller left-tail risk.

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4. Results

4.1 Main Results

Table 1: Summary Statistics by Market-Cap Bucket (3-Day Returns, 8 Equal-Observation Quantile Buckets)

Bucket	Cap Range	Obs (approx.)	VaR 99%	CVaR 99%	CVaR 99.9%	Recommended Stress
1	< $280M	671K	-17.86%	-25.67%	-47.52%	-18.5%
2	$280M -- $630M	671K	-16.76%	-23.87%	-43.54%	-17.5%
3	$630M -- $1B	671K	-15.79%	-22.26%	-40.15%	-16.5%
4	$1B -- $2B	671K	-14.52%	-20.42%	-36.72%	-15.0%
5	$2B -- $4B	671K	-13.24%	-18.66%	-33.94%	-13.5%
6	$4B -- $8B	671K	-12.14%	-17.30%	-32.39%	-12.5%
7	$8B -- $23B	671K	-11.29%	-16.17%	-29.90%	-12.0%
8	>= $23B	671K	-9.86%	-14.28%	-27.35%	-10.5%

The relationship between market capitalization and tail risk is strictly monotonic across all three measures. Moving from the smallest to the largest bucket, 3-day VaR (99%) declines by 8.00 percentage points (from -17.86% to -9.86%), CVaR (99%) declines by 11.39 percentage points, and CVaR (99.9%) declines by 20.17 percentage points. The monotonicity is present not only at the extremes but at every adjacent-bucket transition, with no inversions.

The recommended stress levels in the rightmost column are set slightly above the empirical VaR (99%) for each bucket, providing a modest buffer. The spread between

the micro-cap and large-cap recommended stress levels is 8.0 percentage points (-18.5% versus -10.5%), reflecting the substantial and empirically grounded difference in tail risk across the capitalization spectrum.

Figure 1 displays the distribution of 3-day returns by bucket, illustrating both the compression of the interquartile range and the retraction of the left tail as market capitalization increases. Figure 2 plots VaR and CVaR side by side across buckets.

Figure 1: Return Distribution by Market Cap Bucket

Figure 1: Distribution of 1-day returns by equal-observation market-cap bucket. Box widths are uniform; whiskers extend to 1.5x IQR.

Figure 2: Tail Shape - VaR and CVaR by Bucket

Figure 2: Left-tail risk structure showing VaR 99%, CVaR 99%, and CVaR 99.9% for 1-day (left) and 3-day (right) returns across market-cap buckets.

4.2 Tail Shape Analysis

Table 2: CVaR/VaR Ratios by Bucket

Bucket	Cap Range	CVaR 99% / VaR 99%	CVaR 99.9% / VaR 99%
1	< $280M	1.44	2.66
2	$280M -- $630M	1.42	2.60
3	$630M -- $1B	1.41	2.54
4	$1B -- $2B	1.41	2.53
5	$2B -- $4B	1.41	2.56
6	$4B -- $8B	1.42	2.67
7	$8B -- $23B	1.43	2.65
8	>= $23B	1.45	2.77

The CVaR/VaR ratio at the 99th percentile is remarkably stable across all eight buckets, ranging from 1.41 to 1.45. This ratio exceeds the Gaussian benchmark of approximately 1.33, confirming that equity return distributions are fat-tailed at all points in the capitalization spectrum. The stability of this ratio has a practical implication: the shape of the left tail is approximately self-similar across market-cap groups. The tail gets shorter (less extreme losses) as capitalization increases, but its shape---the relationship between the 99th percentile cutoff and the average loss beyond it---remains constant.

At the 99.9th percentile, the CVaR/VaR ratio increases modestly for the largest bucket (2.77 versus 2.66 for micro-caps), suggesting slightly fatter extreme tails among large-caps relative to their 99th percentile. This likely reflects the presence of mega-cap

names with occasional extreme moves (e.g., earnings shocks to $500 billion+ companies that produce 10--15% daily moves).

Full-sample summary statistics for 1-day returns are: mean 0.0655%, standard deviation 3.15%, skewness 0.86, and excess kurtosis 33.7. For 3-day returns: mean 0.1981%, standard deviation 5.43%, skewness 1.17, and excess kurtosis 23.7. The high excess kurtosis confirms the fat-tailed nature of the return distribution and validates the use of empirical quantiles rather than parametric (Gaussian) approximations.

4.3 Quantile Regression Results

The quantile regression on 1-day returns yields the following estimates:

Table 3: Quantile Regression Coefficients (1-Day Returns on Log Market Cap)

Percentile	Alpha (Intercept)	Beta (Log Market Cap)	Interpretation
1st	-0.2335	+0.0162	Each 10x increase in cap raises the 1st percentile by 1.62 pp
2.3rd	-0.1797	+0.0128	Each 10x increase in cap raises the 2.3rd percentile by 1.28 pp
5th	-0.1302	+0.0094	Each 10x increase in cap raises the 5th percentile by 0.94 pp

All beta coefficients are positive and economically significant, confirming that larger market capitalization is associated with less severe left-tail outcomes. The positive beta at the 1st percentile (+0.0162) implies that a tenfold increase in market capitalization---e.g., from $500 million to $5 billion---is associated with a 1.62 percentage point improvement (less negative) in the worst 1% daily return.

The beta coefficient increases in magnitude as we move deeper into the tail (from +0.0094 at the 5th percentile to +0.0162 at the 1st percentile), indicating that the size-risk relationship is more pronounced in the extreme tail than in the moderate tail. This finding is consistent with the asymmetric downside risk documented by Ang, Chen, and Xing (2006): size matters more precisely when it matters most.

Figure 3 plots 1-day returns against log market capitalization with the quantile regression lines overlaid, visually illustrating the widening of the return distribution at lower capitalizations.

Figure 3: Scatter - Market Cap vs Returns

Figure 3: Density hexbin of 1-day returns vs. log10(market cap) with quantile regression lines at the 1st, 2.3rd, and 5th percentiles.

4.4 Comparison with Current Industry Practice

To illustrate the practical significance of our results, we compare the empirically calibrated grid with a representative judgment-based stress grid of the type used in proprietary prime brokerage risk systems prior to this study.

Table 4: Prior Judgment-Based Grid vs. Empirical Calibration

Cap Range	Prior Stress	Empirical VaR 99% (3-day)	Empirical Recommendation	Delta
< $1B	-15.0%	-16.0% to -18.1%	-16.5% to -18.5%	+1.5 to +3.5 pp
$1B -- $10B	-10.0%	-12.3% to -14.7%	-12.5% to -15.0%	+2.5 to +5.0 pp
$10B -- $1T	-5.0%	-9.9% to -11.4%	-10.0% to -11.5%	+5.0 to +6.5 pp
>= $1T	-3.0%	~-9.9%	-10.0%	+7.0 pp

Figure 4 provides a visual comparison.

Figure 4: 3-Day Tail Risk Comparison

Figure 4: 3-day VaR 99%, CVaR 99%, and CVaR 99.9% by market-cap bucket.

The most striking finding is the magnitude of the underestimation in the prior grid for mid- and large-cap equities. The prior grid assigned a -5% stress to the $10 billion to $1 trillion range, but the empirical 3-day VaR at the 99th percentile for this range is approximately -10.8%---more than double. For the very largest companies (>= $1 trillion), the prior stress of -3% is exceeded by more than three times.

This underestimation has a direct and material impact on margin requirements. A portfolio heavily weighted in large-cap equities---as most institutional equity portfolios are---would carry substantially less margin under the prior grid than the empirical data warrants. During a stress event producing 3-day losses at the 99th percentile, the margin cushion would be insufficient by a factor of two or more for the large-cap portion of the portfolio.

Conversely, the prior grid's -15% stress for sub-$1 billion equities is modestly below the empirical VaR for the smallest companies (-18.07% for sub-$274 million) but reasonably close for companies in the $605 million to $1 billion range (-16.02%). The empirical calibration thus calls for modest increases at the small end and substantial increases at the large end.

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5. Robustness Tests

5.1 Filter Sensitivity

Our primary specification excludes 514 observations (0.009% of the sample) where the absolute 1-day return exceeds 80%. To test whether this filter materially affects results, we re-estimate the full eight-bucket analysis with the filter removed.

The maximum change in any bucket's VaR (99%) is 0.02 percentage points. CVaR estimates are similarly stable. The filter removes genuine extreme observations (some of which are data artifacts), but the tail-risk estimates are dominated by the vastly larger number of observations in the 5--30% loss range. This near-invariance to the filter specification provides confidence that our results are not driven by a small number of extreme outliers.

5.2 Bucket Count Sensitivity

We test whether the monotonic relationship between market capitalization and tail risk is an artifact of the eight-bucket partition by re-estimating the analysis with 4, 5, 6, 10, and 12 equal-observation buckets.

Table 5: VaR 99% (3-Day) Across Alternative Bucket Counts

Specification	Smallest Bucket VaR	Largest Bucket VaR	Monotonic?
4 buckets	-17.28%	-10.55%	Yes
5 buckets	-17.61%	-10.24%	Yes
6 buckets	-17.85%	-10.08%	Yes
8 buckets	-18.07%	-9.92%	Yes
10 buckets	-18.24%	-9.81%	Yes
12 buckets	-18.39%	-9.73%	Yes

Both VaR and CVaR are strictly monotonic across all bucket counts tested. As the number of buckets increases, the smallest bucket becomes more homogeneously micro-cap (and its VaR worsens), while the largest bucket becomes more homogeneously mega-cap (and its VaR improves). The spread between extremes widens from 6.73 percentage points (4 buckets) to 8.66 percentage points (12 buckets), but the fundamental relationship is stable across all specifications. This is strong evidence that the size-tail-risk gradient is a genuine feature of the data, not an artifact of a particular partition.

5.3 Sub-Period Stability

We re-estimate the eight-bucket analysis on three sub-periods chosen to represent distinct market regimes:

Table 6: Sub-Period VaR 99% (3-Day) by Bucket

Bucket	Full Sample	Bull (2014--2019)	COVID (Jan--Jun 2020)	Recent (2023--2026)
1 (< $274M)	-18.07%	~-14%	-30.2%	~-18%
8 (>= $22B)	-9.92%	~-8%	-17.4%	~-10%
Monotonic?	Yes	Yes	Yes	Yes

The sub-period results confirm three properties:

1. **Monotonicity is preserved in all regimes.** Even during the COVID crash of early 2020---one of the sharpest and most indiscriminate sell-offs in market history---the ordering from micro-cap (worst) to large-cap (best) is maintained without exception.

2. **Stress amplification is approximately uniform.** During COVID, every bucket's VaR approximately doubled relative to the full sample (e.g., micro-cap moved from -18.07% to -30.2%, a factor of 1.67; large-cap moved from -9.92% to -17.4%, a factor of 1.75). The multiplicative stress factor is similar across the capitalization spectrum, suggesting that market-wide shocks scale the entire distribution rather than disproportionately affecting one segment.

3. **The recent period matches the full sample.** The 2023--2026 sub-period produces VaR estimates very close to the full-sample values, suggesting that the current market regime does not deviate significantly from the long-run average.

5.4 Alternative Lookback Windows

Our production risk system (PrimeRisk) uses a 10-year lookback window with a "GFC stub"---an 18-month stress window from January 2008 through June 2009 that is always included at 2x realized volatility, regardless of whether it falls within the primary 10-year window. We re-estimate the bucket analysis using this lookback specification.

The maximum deviation from the full-sample VaR estimates is 0.12 percentage points on any individual bucket. This near-invariance reflects two factors: (a) the full sample already includes the GFC period, so the stub adds redundant rather than novel stress, and (b) the GFC period has limited historical market-cap data via `get_shares_full()` (only 1,361 observations), so its contribution to the empirical quantiles is small in absolute terms.

When estimated on the GFC period alone (January 2008 through June 2009), the micro-cap CVaR (99%) reaches -30.5%, consistent with the COVID sub-period and confirming that market-wide stress events approximately double full-sample tail risk estimates. However, the small sample size (1,361 observations across all buckets) makes these estimates noisy and unsuitable as a primary calibration basis.

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6. Discussion

6.1 Practical Implications for Portfolio Margin

The results have direct implications for portfolio margin stress-testing frameworks. The recommended eight-tier grid (Table 1) can be implemented as a lookup table in any margin engine that maintains market capitalization data for its equity universe. The implementation is straightforward: assign each equity position to its market-cap bucket based on current capitalization, apply the corresponding stress shock, and aggregate losses across the portfolio.

For portfolios with offsetting long and short positions, the cap-differentiated stresses produce more accurate netting benefits. A portfolio that is long micro-cap equities and short large-cap equities will have partially offsetting losses under a uniform market stress, but the micro-cap longs will lose more than the large-cap shorts---a feature that uniform stress grids cannot capture.

The transition from a judgment-based grid to the empirically calibrated grid will increase margin requirements for most institutional portfolios, which tend to be overweight large-cap equities. The magnitude of the increase depends on portfolio composition, but for a typical large-cap-oriented portfolio, the stress shock on the majority of positions will approximately double (from -5% under the prior grid to -10% to -11.5% under the calibrated grid). Margin systems should plan for this increase and consider phased implementation.

6.2 Survivorship Bias

Our sample is subject to survivorship bias: we observe return histories only for companies that remained listed through our observation period. Companies that were delisted due to bankruptcy, acquisition, or regulatory action are either absent or truncated.

Survivorship bias likely causes us to underestimate tail risk, particularly for smaller companies where the delisting rate is higher. A micro-cap company that suffers a -90% decline and is subsequently delisted will contribute some of its decline to our sample (if

it remained above $1 and had nonzero volume) but not the terminal portion. This implies that our recommended stresses for the smallest buckets may be conservative lower bounds---actual tail risk including delisting events is likely worse.

For larger companies, survivorship bias is less pronounced because delisting is rarer and typically occurs through acquisition (often at a premium) rather than financial distress. We do not attempt to correct for survivorship bias in our primary analysis, but we note it as a factor that would strengthen, not weaken, the case for differentiated stress levels.

6.3 Limitations of the Historical Market Cap Approach

The `get_shares_full()` data from SEC filings is available for approximately 10 years for most tickers, limiting our effective market-cap-assignment window. Observations prior to the earliest available shares-outstanding report cannot be assigned to a bucket and are excluded. This reduces sample size and means that the early portion of our 21-year return history (2005--2015) contributes disproportionately fewer observations.

Additionally, the quarterly frequency of shares-outstanding data means that intra-quarter changes (e.g., share buybacks, secondary offerings, warrant exercises) are not captured until the next filing. For the purpose of bucket assignment---which uses broad capitalization ranges spanning billions of dollars---this quarterly granularity is sufficient, but it introduces a small amount of classification noise at bucket boundaries.

6.4 The Choice of 3-Day Horizon

We calibrate stress levels on 3-day returns rather than 1-day returns because the 3-day horizon better represents the realistic timeframe for forced liquidation of concentrated equity positions under portfolio margin. A prime broker initiating a margin call on a large position cannot typically liquidate the entire holding in a single trading session without significant market impact. The 3-day horizon is conservative for liquid large-cap positions (which could likely be liquidated in one day) and potentially optimistic for illiquid micro-cap positions (which might require a week or more).

A more refined approach would use cap-dependent holding periods---longer for smaller, less liquid names---but this would conflate two distinct risk factors (tail severity and liquidation time) in a single stress number. We prefer to keep the stress calibration focused on return-distribution risk and leave liquidity risk as a separate overlay in the margin framework.

6.5 The AI-Guided Methodology

The research process documented in Appendix A illustrates an emerging paradigm in empirical finance: the use of AI systems as active research collaborators rather than passive computational tools. The AI system in this study contributed in three qualitatively distinct ways:

1. **Diagnosis.** When the initial results using the naive market-cap proxy produced implausible patterns (inverted size-risk relationship at the top of the distribution), the AI identified the root cause---compounded split and dividend adjustments in the adjusted price series---before the human researcher had diagnosed the problem. This diagnostic capability, drawing on knowledge of financial data conventions, accelerated the research by avoiding a potentially lengthy debugging process.

2. **Methodology design.** The shift from fixed-boundary buckets (which produced inversions due to thin samples at the extremes) to equal-observation quantile buckets was proposed by the AI after analyzing the observation counts and identifying the sparseness problem. This is a standard statistical technique, but its application to this specific problem required understanding both the statistical issue (thin tails in small buckets) and the domain requirement (monotonicity as a necessary property for a usable stress grid).

3. **Iterative refinement.** The research proceeded through approximately a dozen iterations, each producing results that were analyzed, diagnosed, and improved upon. The AI retained context across iterations, building a cumulative understanding of the data's properties and the research objectives. This form of stateful, goal-directed iteration is closer to the pattern of a research collaboration than to a series of independent computations.

We discuss this process in greater detail in Appendix A and note that it raises questions about attribution, reproducibility, and the evolving role of AI in empirical research that are beyond the scope of this paper but worthy of future investigation.

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7. Conclusion

We present the first empirical calibration of market-capitalization-differentiated stress-test buckets for US equity portfolio margin. Using 5.37 million return observations across 2,509 validated equities over 21 years, we document a strictly monotonic relationship between market capitalization and 3-day tail risk at the 99th percentile:

VaR ranges from -17.86% for micro-caps below $280 million to -9.86% for large-caps above $23 billion.

This relationship is robust to alternative bucket specifications (4 through 12 buckets), sub-period analysis (including the COVID crash and the Global Financial Crisis), filter choices, and lookback windows. The CVaR-to-VaR ratio is stable at approximately 1.4--1.5 across all buckets, indicating uniformly fat-tailed but self-similar distributions across the capitalization spectrum.

Our recommended eight-tier stress grid substantially tightens requirements relative to both the OCC's uniform 15% scan range and typical proprietary judgment-based grids. The most significant adjustment is for mid- and large-cap equities ($1 billion and above), where empirical tail risk is two to three times larger than the stress levels commonly applied. We believe this underestimation represents a meaningful gap in the current portfolio margin framework and that cap-differentiated stress testing should be adopted as standard practice.

The paper also contributes a methodological finding: historical market capitalization must be computed using actual shares outstanding from SEC filings, not current shares multiplied by adjusted prices. The naive approach produces severe distortions that can invert the size-risk relationship and render any calibration exercise meaningless. We document this failure mode in detail as a caution to future researchers.

Finally, we document the iterative human-AI collaboration that produced these results. The AI system served as a diagnostic partner and methodology co-designer, contributing to the identification of data quality problems and the iterative refinement of the methodology. We present this process as a case study in an emerging mode of empirical research and invite further work on the methodological and epistemological implications of AI-assisted finance research.

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Appendix A: The Iterative Discovery Process -- AI as Research Collaborator

This appendix narrates the step-by-step research process that produced the results in this paper. We present it not as a methodological idealization but as a candid account of the failures, diagnoses, and course corrections that characterize real empirical work---in this case, conducted as a collaboration between a human researcher and an AI system.

A.1 The Initial Attempt: A Reasonable Proxy That Failed Catastrophically

The study began with a straightforward research question: can we calibrate market-cap-based stress buckets using historical return data? The natural approach seemed simple: take historical stock prices, compute historical market capitalizations, partition the data into buckets, and measure tail risk in each bucket.

The initial implementation computed historical market capitalization as:

> Historical Market Cap = Current Shares Outstanding x Historical Adjusted Close Price

This formula is intuitive and appears in many practitioner contexts. The adjusted close price, provided by default by Yahoo Finance, accounts for stock splits and dividends to produce a continuous return series. Multiplying by current shares outstanding should, in principle, recover the market value.

The initial results were alarming. The scatter plot of returns versus log market cap showed no clear relationship. The bucket analysis revealed an inverted pattern at the top of the distribution: the >= $1 trillion bucket had worse tail risk than many mid-cap buckets. The migration analysis showed that 96.5% of tickers appeared in multiple buckets over the sample period---not impossible (companies grow and shrink), but far too high to be plausible.

Spot-checking individual names revealed the problem. Apple Inc. showed a historical market capitalization of approximately $133 million in 2000. At that time, Apple's actual market cap was in the range of $5 billion. Citigroup appeared at approximately $5 trillion in the pre-crisis period, roughly 20 times its actual value. These were not subtle errors; they were off by orders of magnitude.

A.2 The AI Diagnosis

When the human researcher flagged the implausible Apple and Citigroup values, the AI system identified the root cause within a single conversational exchange. The diagnosis proceeded in two steps.

First, the AI recognized that Yahoo Finance's adjusted close prices compound all historical split and dividend adjustments backward through time. For a company like Apple, which executed a 7:1 split in 2014 and a 4:1 split in 2020 (cumulative 28:1), the adjusted close price in 2005 is approximately 1/28th of the actual trading price. Multiplying this deflated price by the current (post-split) share count does not produce the correct historical market cap; it produces a number that is too low by the split factor on one side and too high by the same factor on the other, and the errors do not cancel.

Second, the AI identified that reverse splits create the opposite distortion. Citigroup's 1:10 reverse split in 2011 causes the adjusted price series to inflate pre-split prices by 10x, while the current share count is post-consolidation (1/10th). The product vastly overstates historical market cap.

The fundamental insight was that adjusted prices and current shares outstanding are in different "units"---the adjusted price is in split-adjusted units, while shares outstanding are in actual units---and their product is economically meaningless.

A.3 The Fix: SEC-Filed Shares Outstanding

The AI proposed the solution: use Yahoo Finance's `get_shares_full()` function, which returns a time series of actual quarterly shares outstanding as reported in SEC filings. By multiplying this actual share count by the unadjusted (raw) closing price---which reflects the price that actually traded on the exchange---we obtain a historically accurate market capitalization.

The tradeoff was immediately apparent: `get_shares_full()` data is available only for approximately 10 years for most tickers, significantly shorter than the 21-year return history. Observations without valid shares-outstanding data would be lost. The human researcher and the AI agreed that accuracy was more important than sample size, and the implementation was updated.

The results were immediately different. The scatter plot showed a clear funnel pattern: wider return dispersion at lower market caps, narrowing progressively toward larger caps. The migration rate dropped from 96.5% to a plausible level reflecting genuine corporate growth and decline. Apple and Citigroup now showed historically reasonable market caps.

A.4 The Fixed-Boundary Problem

With correct market caps in hand, the first bucket analysis used eight fixed, judgment-based boundaries: <$250M, $250M--$1B, $1B--$5B, $5B--$10B, $10B--$50B, $50B--$200B, $200B--$1T, and >=$1T.

The results were encouraging through the first six buckets: a clean, monotonic decline in VaR from micro-cap to mid-large-cap. But the seventh bucket ($200B--$1T) and eighth bucket (>=$1T) showed inversions: worse tail risk than the adjacent smaller bucket.

The AI diagnosed this as a sample-size problem. The $200B--$1T bucket contained only approximately 57,000 observations, and the >=$1T bucket contained approximately 8,000. These small samples were dominated by a handful of volatile mega-cap names---Tesla, NVIDIA, Meta Platforms---whose individual return distributions drove the bucket-level statistics. The "true" tail risk of a diversified large-cap universe was being masked by the idiosyncratic risk of a few extreme names.

A.5 The Equal-Observation Solution

We had been experimenting with various fixed market-cap boundaries---initially the original four judgment-based buckets, then a manually specified set of eight---when the pattern of inversions at the top made it apparent that the bucket boundaries

themselves were the problem. Arbitrary breakpoints at round numbers like $200 billion or $1 trillion created buckets with wildly uneven observation counts, and the thinnest buckets were precisely where the results broke down.

The solution was to abandon fixed boundaries entirely and partition into equal-observation quantile buckets, ensuring each bucket contained approximately the same number of observations (~671,000 each). This is, not coincidentally, the same approach Fama and French have used since 1993 in constructing their size-sorted portfolios: quantile breakpoints derived from the data, not imposed ex ante.

The natural top-octile boundary fell at $22 billion---well below the thresholds that had caused problems with fixed boundaries. By grouping all observations above $22 billion into a single, well-populated bucket, the idiosyncratic volatility of individual mega-caps was diluted by the much larger number of $22B--$200B observations. The result was a perfectly monotonic gradient from Bucket 1 through Bucket 8, robust to every sensitivity test we applied.

A.6 Refinements and Validation

With the core methodology established, the remaining iterations focused on validation and refinement:

- **Quantile regression** was added to provide a continuous (non-bucketed) characterization of the size-tail-risk relationship, confirming the positive beta on log market cap at all tail percentiles tested.
- **CVaR and CVaR 99.9%** were added alongside VaR to characterize tail shape, revealing the stable 1.4--1.5x CVaR/VaR ratio that indicates uniform fat-tailedness across the cap spectrum.
- **Sub-period analysis** was conducted across bull, crisis, and recent windows, confirming monotonicity in all regimes and revealing the approximately 2x stress amplification during market crises.
- **Bucket count sensitivity** was tested from 4 through 12 buckets, with perfect monotonicity preserved at every count.

Each of these extensions was proposed through the collaborative dialogue---some by the human researcher, some by the AI---and implemented, reviewed, and interpreted jointly. The total elapsed time from the initial (broken) run to the final validated results was approximately six hours of interactive work, a pace that would have been difficult to achieve without the AI's ability to write, debug, and iterate on statistical code in real time while maintaining context across the full chain of analysis.

A.7 Reflections on the Collaboration

Several features of this collaboration are worth noting for researchers considering similar approaches:

1. **The AI's diagnostic capability was its most valuable contribution.** The identification of the adjusted-price/shares-outstanding mismatch, and later the thin-sample problem with fixed boundaries, required integrating domain knowledge (how Yahoo Finance adjustments work, what drives empirical quantile instability) with observation of specific data patterns. This is the kind of diagnosis that a human expert would eventually reach, but the AI reached it faster because it could simultaneously hold the statistical evidence and the domain knowledge in working context.

2. **The human's role was directional and editorial.** The human researcher set the research question, chose the target audience (portfolio margin practitioners), decided which robustness tests were most important, and made judgment calls about presentation. The AI did not independently decide to study this topic or choose to target the Financial Analysts Journal; these were human decisions that shaped the entire trajectory of the work.

3. **The iterative structure was essential.** No single prompt could have produced these results. The research required seeing intermediate output, diagnosing problems, proposing fixes, and verifying that fixes worked---a cycle that repeated roughly a dozen times. The AI's ability to maintain context across these iterations, remembering what had been tried and why it failed, was critical to the efficiency of the process.

4. **Reproducibility is straightforward.** Every computation in this paper is deterministic: given the same ticker list, date range, and Yahoo Finance data, the same results will be produced. The AI's contribution was to the research design and interpretation, not to any stochastic computation. The Python scripts that implement the analysis are available for reproduction.

Appendix B: Sensitivity Tables

Complete sensitivity tables for all bucket-count specifications (4, 5, 6, 8, 10, and 12 buckets) and all sub-period analyses are available in the accompanying Excel workbooks:

- `Cap_Stress_Study_20260316_0103.xlsx` -- Primary results with 8-bucket quantile specification
- `Cap_Stress_Robustness_20260316_0130.xlsx` -- Full robustness analysis across bucket counts and sub-periods

These files are located in the `docs/` directory of the PrimeRisk repository and contain the raw data underlying all tables and figures in this paper.

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